HEAT EXCHANGE IN MOTIONLESS VAPOR CONDENSATION ON A PLATE IMMERSED IN A GRANULAR BED

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UDC 536.423.4:536.
242:532.546

A model accounting for the boundary zone of elevated thermal resistance at the wall and the convective heat transport in a fill is proposed.

The problem of vapor condensation in capillary-porous solids (CPS) and at surfaces embedded in a granular bed is of great interest for the chemical technology, geothermodynamics (for geothermal heat extraction), heat techniques of increasing oil and gas production by pumping steam into the critical zone of the stratum, and the design and calculation of heat pipes and other parts of power plants and devices.

Calculation of the heat exchange during vapor condensation in such structures is complicated by the many factors determining heat transfer in such processes. They include the packing geometry, the framework thermal conductivity, the effect of capillary forces, and the thermohydrodynamics of constrained flow around the fill zone and CPS's, which in the final analysis affect the laws governing the effective coefficients of thermal conductivity and viscosity.

The present paper is concerned with the heat exchange during condensation of motionless vapor on an inclined smooth plate, embedded in a granular bed. Besides its separate interest for various technological processes, this problem can be considered as a certain approximation in simulating heat exchange during condensation in a CPS, which would make it possible to analyze the effect of a number of important factors on heat transfer.

The published papers [1, 2] provide only very simple models of the condensation process in granular beds. Thus, a self-similar solution is given in [1] for the problem of heat exchange in motionless vapor condensation on an inclined plate in a fill under the assumption that the effective thermal conductivity coefficient across the condensate film is constant for boundary conditions of the first kind pertaining to the film. It is considered that the filtration rate of the film obeys the Darcy law. For the limiting cases, the ordinary differential equation obtained can be integrated in finite form. An approximate expression for the local heat transfer coefficient is proposed on the basis of these solutions.

An integral method for heat exchange calculations is given in [2] under the same assumptions, but with additional requirements, similar to Nusselt's problem for motionless vapor condensation on a smooth plate. A comparison with the authors' experimental data (these data are the only ones known in the literature) has shown satisfactory agreement between the theoretical and experimental results for some of the operating parameters.

At the same time, many of the tabular data given in [2] indicate that the heat exchange law deviates considerably from the proposed models [1, 2], which has not been explained within the framework of these models.

We propose here a model which accounts for the convective component in heat transfer and the presence of a boundary zone in the downward running film, where the effective thermal conductivity coefficient differs substantially from its value in the main body of the film (which is primarily due to the different porosity values of the granular medium in the flow core and in the wall zone).

Institute of Thermophysics, Siberian Branch, Academy of Sciences of the USSR, Novosibirsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 54, No. 2, pp. 181-188, February, 1988. Original article submitted March 23, 1986.

0022-0841/88/5402-0119$12.50

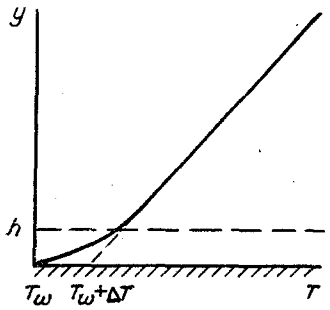

Fig. 1

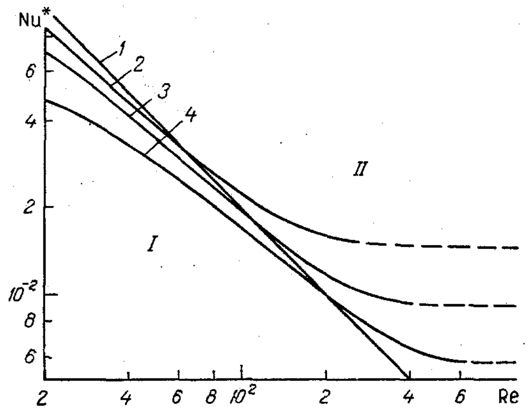

Fig. 2

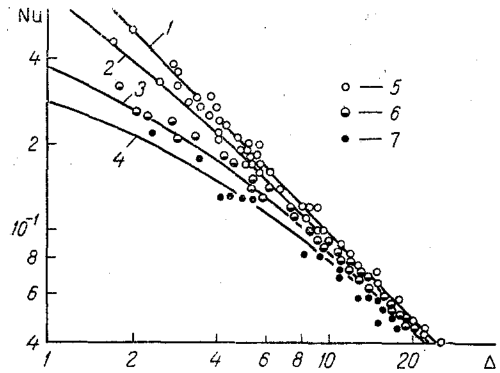

Fig. 3

Fig. 1. Mean temperature profile in the liquid film near a solid wall.

Fig. 2. Dependence of Nu* on the Re number for $\tilde{p}$ = 1.1. 1) Re* = 1.5; 2) 3.8; 3) 9.7; 4) 11.2; the dashed curves represent the self-similar solution [1] for the corresponding Re* numbers.

Fig. 3. Effect of the film thickness Δ on the heat exchange. 1-4) Calculation based on (21) for $\tilde{p}$ = 0, 0.5, 1.7, and 2.6, respectively; 5-7) experimental data from [2] (p ≤ 0.9, p = 1-1.9, and p = 2-2.9, respectively).

We assume that the outer boundary of the film is at the saturation temperature T_s, there is no friction between the phases, and the physical characteristics of the liquid do not vary. The plate temperature T_w is kept constant. Heat transfer is realized due to the effective thermal conductivity of the liquid across the film and by convective transport in the longitudinal direction.

According to Darcy's law, the liquid velocity in relatively "thick" (in comparison with the radius R of grains in the fill) films is assumed to be

$$U = \frac{\Pi}{\mu}(\rho_L - \rho_g)\, g \cos\varphi. \tag{1}$$

The heat transfer equation and the boundary conditions are written in the following form:

$$U\frac{\partial T}{\partial x} = a_e\frac{\partial^2 T}{\partial y^2}, \tag{2}$$

$$y = \delta, \quad T = T_s, \tag{3}$$

$$y = 0, \quad T = T_w, \tag{4}$$

where a_e is the effective thermal diffusivity coefficient.

The film thickness δ is determined from the equation of balance of the condensate mass,

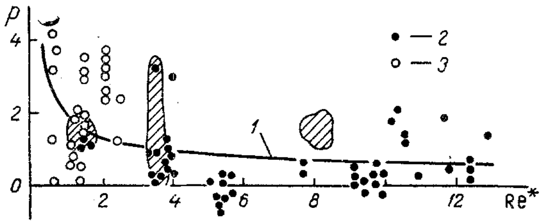

Fig. 4. Parameter $\tilde{p}$ as a function of the Reynolds number Re* for flow around a fill element. 1) Calculation based on expression (24) for c = 2.0; 2) sand, d = 0.8 mm; 3) sand, d = 0.5 mm. The hatched areas represent the data on heat exchange for a fill consisting of Nichrome pellets with d = 0.5 mm.

$$d(\rho_L U\delta) = (q/r)\,dx. \tag{5}$$

The measured film temperature profile in the fill [2] indicates the presence of a thin zone with the thickness h at the plate surface where the slope of the temperature profile of the condensate film differs substantially from its slope in the main body of the film. This indicates that the values of the heat transport coefficient differ considerably from the effective values in the outer part of the film. However, this fact was neglected entirely in [1, 2].

The existence of a layer with elevated thermal resistance at the surface of solids immersed in dispersion systems was considered in calculating external, nonstationary heat exchange in fluidized systems [3, 4] and also in problems of heat and mass exchange in filtration through immobile granular beds (for instance, [5]). The presence of a thin zone of elevated thermal resistance at the surface of a fill-embedded solid that is at a certain given temperature was taken into account in [6] by means of boundary conditions of the third kind at this surface. The authors of that paper assumed a zone thickness equal to the grain diameter d in the fill and a parabolic profile of the probability density distribution of grain centers in the surface zone.

We shall also use boundary conditions of the third kind in order to take into account the wall zone in the condensate film. At the distance y ~ h in the main body of the flow, the temperature profile is given in the following form (Fig. 1):

$$T(y) = T_w + \Delta T + \Gamma y, \quad \Gamma = (dT/dy)_{y\approx h}. \tag{6}$$

In contrast to [6], we shall use a somewhat simplified method to determine the relationship between the temperature deficiency ΔT and the gradient (dT/dy) for $y \to 0$. By representing the temperature profile in the wall layer in the form of a Taylor series and retaining only terms of the first order, we obtain

$$T(y) = T_w + \Gamma_w y, \quad \Gamma_w = (dT/dy)_{y\to 0}. \tag{7}$$

From the equality of thermal fluxes at the boundary $y \approx h$, we have

$$\Gamma = (\lambda_w/\lambda_e)\,\Gamma_w,$$

whence, with an allowance for relationships (6) and (7) for $y \to h$, it follows that

$$\Delta T = h\Gamma(\lambda_e/\lambda_w - 1).$$

Then, boundary condition (4) can be rewritten thus:

$$T - p\frac{dT}{dy} = T_w \quad \text{for} \quad y = 0, \tag{8}$$

where $p = h(\lambda_e/\lambda_w - 1)$.

The form of boundary condition (8) is identical to that obtained in [6]. They differ from each other only by the specific form of writing the parameter p. The proposed representation of the parameter p is convenient for estimating it by means of simple semiempirical models and for processing experimental data on the basis of experimental values of the effective thermal conductivity coefficient λ_e within the film and the temperature distribution across the film.

Thus, introducing the notation $\Theta = (T - T_s)/(T_w - T_s)$ and $\kappa = a_e/U$, we state the problem of heat exchange during motionless vapor condensation in a fill in the following manner:

$$\frac{\partial \Theta}{\partial x} = \varkappa \frac{\partial^2 \Theta}{\partial y^2}, \tag{9}$$

$$\Theta - p \frac{\partial \Theta}{\partial y} = 1 \quad \text{for} \quad y = 0, \tag{10}$$

$$\Theta = 0 \quad \text{for} \quad x = 0, \tag{11}$$

$$\Theta = 0 \quad \text{for} \quad y = \delta. \tag{12}$$

Let us introduce the integral-mean film thickness $\tilde{\delta}$ over the plate's length L:

$$\overline{\delta} = L^{-1} \int_0^{\delta} \delta(x)\, dx.$$

The balance equation (5) assumes the following form:

$$\rho U \overline{\delta} = qL/(2r). \tag{13}$$

Instead of boundary condition (12), we have

$$\Theta = 0 \quad \text{for} \quad y = \overline{\delta}. \tag{14}$$

We represent $\Theta(x, y)$ as a superposition of two functions:

$$\Theta(x, y) = v(y) + u(x, y),$$

where v(y) is determined by solving the problem

$$\frac{d^2 v}{dy^2} = 0, \quad \left(v - p \frac{dv}{dy}\right)_{y=0} = 1, \quad v|_{y=\overline{\delta}} = 0, \tag{15}$$

while u(x, y) is the solution of the equation

$$\frac{\partial u}{\partial x} = \varkappa \frac{\partial^2 u}{\partial y^2} \tag{16}$$

for the boundary conditions

$$\left(u - p \frac{\partial u}{\partial y}\right)_{y=0} = 0, \quad u|_{y=\overline{\delta}} = 0 \tag{17}$$

and the initial condition

$$u|_{x=0} = -v(y). \tag{18}$$

The solution of problem (15) is given by

$$v(y) = (\overline{\delta} - y)/(\overline{\delta} + p).$$

For the problem (16)-(18), we correspondingly have

$$u(x, y) = \sum_{n=1}^{\infty} c_n [\cos(\lambda_n y) + (\lambda_n p)^{-1} \sin(\lambda_n y)] \exp(-\lambda_n^2 \varkappa x),$$

where λ_n are the eigenvalues constituting the roots of the transcendental equation

$$\operatorname{tg}(\lambda_n \overline{\delta}) = -\lambda_n p.$$

From the initial condition, we obtain

$$c_n = -2p/[\overline{\delta}(\lambda_n^2 p^2 + 1) - p].$$

Thus, the general solution of the problem defined by (9)-(13) and (14) is written as follows:

$$\Theta(x, y) = \frac{\bar{\delta} - y}{\bar{\delta} + p} - \sum_{n=1}^{\infty} \frac{2p \exp(-\varkappa\lambda_n^2 x)}{\bar{\delta}(\lambda_n^2 p^2 + 1) - p} [\cos(\lambda_n y) + (\lambda_n p)^{-1} \sin(\lambda_n y)]. \qquad (19)$$

Hence, for the thermal flux averaged along the plate, we find the following with an allowance for $\exp(-\kappa\lambda_n^2 x) << 1$:

$$\bar{q} = \frac{\lambda_e \Delta T}{\bar{\delta} + p} + \frac{2}{\varkappa L} \sum_{n=1}^{\infty} \frac{\lambda_e \Delta T}{\lambda_n^2 [\bar{\delta}(\lambda_n^2 p^2 + 1) - p]}. \qquad (20)$$

We introduce the following dimensionless numbers and parameters:

$$\mathrm{Re} = \frac{qL}{r\mu}; \quad \mathrm{Nu} = \frac{\alpha R}{\lambda_e}; \quad \mathrm{Pr} = \frac{\nu}{a}; \quad \mathrm{Ar}^* = \frac{g\Pi L}{\nu^2}\left(1 - \frac{\rho_g}{\rho_L}\right)\cos\varphi;$$

$$\mathrm{Nu}^* = \frac{\mathrm{Nu}}{\mathrm{Re}^*}; \quad \mathrm{Re}^* = \mathrm{Ar}^* \frac{R}{L}; \quad \mathrm{Pe}^* = \mathrm{Ar}^* \frac{\nu}{a_e}; \quad \Delta = \frac{\bar{\delta}}{R};$$

$$\tilde{p} = \frac{p}{R}.$$

Then, for the mean film thickness, we obtain from (13) $\Delta = \mathrm{Re}/(2\mathrm{Re}^*)$, while the following holds for the heat-transfer coefficient according to (20):

$$\mathrm{Nu} = \frac{1}{\Delta + \tilde{p}} + 2\mathrm{Pe}^* \sum_{n=1}^{\infty} \lambda_n^{-2} L^{-2} [(1 + \lambda_n^2 R^2 \tilde{p}^2)\Delta - \tilde{p}]^{-1} \qquad (21)$$

or

$$\mathrm{Nu}^* = \frac{2}{\mathrm{Re} + \tilde{p}\,\mathrm{Re}^*} + 4\mathrm{Pe}^* \sum_{n=1}^{\infty} \lambda_n^{-2} L^{-2} [(1 + \lambda_n^2 R^2 \tilde{p}^2)\,\mathrm{Re} - 2\tilde{p}\mathrm{Ar}^*]^{-1}. \qquad (22)$$

Here, the first term characterizes the heat exchange resulting from the thermal conductivity characterized by the effective thermal conductivity coefficient λ_e, while the second term accounts for the contribution of the convective thermal flux.

For $\tilde{p} = 0$ (single-layer model with a constant thermal conductivity λ_e over the cross section of the film) and without consideration of the convective mechanism of heat transfer, we obtain from (22)

$$\mathrm{Nu}^* = 2\mathrm{Re}^{-1}. \qquad (23)$$

This result was obtained in [2] by using the integral method without allowing for convection in the wall zone.

The order of magnitude of the wall zone thickness h is estimated on the basis of dimensionality relationships:

$$\tilde{h} = h/L \sim (\nu/\omega)^{1/2}/L \sim (\nu R/U)^{1/2}/L = \chi\,(R/L)\,(\mathrm{Re}^*)^{-1/2}.$$

Thus, the following holds for $\tilde{p}$:

$$\tilde{p} = p/R = \chi\,(\lambda_e/\lambda_w - 1)\,(\mathrm{Re}^*)^{-1/2}. \qquad (24)$$

Figure 2 shows the behavior of the Nusselt number Nu*, calculated by means of (22). The numerical values of the Re* parameter (the Reynolds number for flow around a fill element) correspond to the experimental values obtained in [2]. Region I in Fig. 2 corresponds to heat exchange resulting primarily from thermal conductivity characterized by the effective coefficients λ_e and λ_w (the value of λ_e was determined experimentally in [2]). Region II is characterized by a significant contribution of convective heat transport.

Analysis of relation (22) shows that the characteristics of the wall zone affect considerably the heat exchange only in region I; it is actually here that considerable differentiation with respect to $\tilde{p}$ values is observed for a fixed Re* number. At the same time, this differentiation is virtually absent in region II.

Comparison with the experimental data from [2] can be performed more conveniently on a Nu vs Δ diagram on the basis of relationship (21) (see Fig. 3). These data are satisfactorily described within the framework of the proposed model for suitable values of the parameter $\tilde{p}$.

The behavior of $\tilde{p}$ is shown in Fig. 4. The $\tilde{p}$ values were obtained by processing the corresponding experimental results from [2]. These data can be approximated by an expression of the type (24) with the proportionality factor $c = \chi(\lambda_e - \lambda_w - 1) = 2\text{-}3$.

Thus, consideration of the boundary layer with elevated thermal resistance makes it possible to predict accurately the behavior, and estimate the value, of the heat transfer coefficient in motionless vapor condensation on an inclined plate embedded in a fill. Nevertheless, the accumulated experimental data are insufficient for drawing unambiguous conclusions concerning, for example, the effect of the thermophysical characteristics of the fill material, which is indicated by the considerable scatter of data on $\tilde{p}$ for $Re^* \simeq 3.5$ (hatched areas in Fig. 4) in attempts at processing the experimental results [2] on the heat transfer in a fill of Nichrome pellets.

Analysis of heat transfer for large Reynolds numbers on the basis of expression (19) is difficult because of poor convergence of the series (this corresponds to the range of small and medium values of the dimensionless group $\kappa x/\bar{\delta}^2$). In connection with this, it is more convenient to use the solution obtained by means of the integral Laplace transform:

$$\Theta(x,\ y) = \Phi^*\left(\frac{y}{2\sqrt{\varkappa x}}\right) - \exp\left(\frac{y}{p} + \frac{\varkappa x}{p^2}\right)\Phi^*\left(\frac{y}{2\sqrt{\varkappa x}} + \frac{\sqrt{\varkappa x}}{p}\right) +$$
$$+ \sum_{n=1}^{\infty}\left\{\Phi^*\left(\frac{2n\bar{\delta}+y}{2\sqrt{\varkappa x}}\right) - \Phi^*\left(\frac{2n\bar{\delta}-y}{2\sqrt{\varkappa x}}\right) - \exp\left(\frac{2n\bar{\delta}+y}{p} - \frac{\varkappa x}{p^2}\right) \times\right.$$
$$\left.\times\ \Phi^*\left(\frac{2n\bar{\delta}+y}{2\sqrt{\varkappa x}} + \frac{\sqrt{\varkappa x}}{p}\right) + \exp\left(\frac{2n\bar{\delta}-y}{p} + \frac{\varkappa x}{p^2}\right)\Phi^*\left(\frac{2n\bar{\delta}-y}{2\sqrt{\varkappa x}} + \frac{\sqrt{\varkappa x}}{p}\right)\right\}.$$

Then, we obtain the following for the thermal flux:

$$q = \lambda_e \Delta T\left[\frac{1}{p}\exp\left(\frac{\varkappa x}{p^2}\right)\Phi^*\left(\frac{\sqrt{\varkappa x}}{p}\right) + \sum_{n=1}^{\infty}\frac{2}{p}\exp\left(\frac{2n\bar{\delta}}{p} + Z^2\right)\Phi^*(D+z)\right].$$

Here $\Phi^* = \frac{2}{\sqrt{\pi}}\int\limits_z^{\infty}\exp(-z^2)\,dz$ is the error function, $z = \sqrt{\kappa x}/p$ and $D = n\bar{\delta}/\sqrt{\kappa x}$.

For large values of the argument, the error function can be approximated with an error whose absolute value is smaller than the last retained term by using the series

$$\Phi^*(z) = \pi^{-1/2}\exp(-z^2)\left(\frac{1}{z} - \frac{1}{2z^3} + \ldots\right).$$

Retaining the first term of the expansion and averaging the thermal flux along the plate, we find

$$\bar{q} = \frac{2\lambda_e\Delta T}{L\sqrt{\pi}}\left\{\left(\frac{L}{\varkappa}\right)^{1/2} + \sum_{n=1}^{\infty}\int\limits_0^L\frac{\sqrt{\varkappa x}\exp(-n^2\bar{\delta}^2/\varkappa x)}{p(n\bar{\delta} + \varkappa x/p)}\,dx\right\}.$$

For relatively "thick" films, where the effect of the wall zone can be neglected, the latter expression for the mean thermal flux becomes

$$\bar{q} = 2\lambda_e\Delta T/\sqrt{\pi\varkappa L},$$

so that the following holds for the heat transfer coefficient:

$$\mathrm{Nu} = \frac{2}{\sqrt{\pi}}(\mathrm{Pe}^*)^{1/2},$$

which coincides with the self-similar solution for an infinitely thick film [1], averaged along the plate, and serves as the asymptotic form of the relationships derived above for heat transfer in the range of small and medium values of the $\kappa x/\bar{\delta}^2$ parameter under the assumption that the wall zone has been neglected.

NOTATION

a, thermal diffusivity in the granular bed; L, plate length; h, wall zone thickness; q, local thermal flux; $\bar{q}$, thermal flux averaged along the plate; r, latent heat of phase transition; R, grain radius; T, T_w, and T_s, temperature, plate temperature, and temperature at the film's outer boundary, respectively; U, filtration rate; x and y, longitudinal and transverse coordinates, respectively; α, heat transfer coefficient; δ and $\bar{\delta}$, local and mean film thickness, respectively; λ_L, λ_w, and λ_e, thermal conductivity of the liquid, the wall zone, and the main body of the film, respectively; ρ_L and ρ_g, densities of the liquid phase and vapor, respectively; Π, permeability of the bed; φ, angle between the longitudinal coordinate x and the gravity acceleration vector g; ω, angular velocity; μ and ν, dynamic and kinematic viscosity of the liquid, respectively.

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PROPAGATION OF VIBRATIONS IN A SUSPENDED GRANULAR BED

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UDC 532.529.5:66.036.5

The influence of the elasticity and relative motion of the continuous medium on the hydrodynamics of a suspended vibrating bed is discussed. A solution is given for the boundary-value problem of small pressure disturbances propagating in the bed. The results are compared with experimental data and calculations based on existing models.

1. PHYSICAL MODEL

The action of vibrations on disperse materials for the purpose of intensifying heat- and mass transfer processes has been utilized for some time now with optimistic results [1, 2]. On the other hand, the theory of vibrofluidization [3-5] is far from complete in either the quantitative or the qualitative aspect. It fails to describe high-frequency (>10 Hz) resonance effects, which have been noted by many researchers, including Kroll [3] and Gutman [4], and which enhance heat and mass transfer significantly at their peak development [2]. The discrepancy with experimental results is an outgrowth of a common practice in the mechanics of fluidized systems (FS's) [6], namely the representation of the gaseous medium as an incompressible fluid, which limits the application of the theory to parameters that support the customary relation between the equilibrium pressure P_0 and its variation p: $P_0 \gg p$. The latter corresponds quite well to "low" ($\Delta p_b \ll P_0$) fluidized beds, but

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Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 54, No. 2, pp. 188-197, February, 1988. Original article submitted October 17, 1986.

0022-0841/88/5402-0125$12.50